

Online Appendix

Limited Cognitive Ability and Selective Information Processing

Benson Tsz Kin Leung¹
July 12, 2019

A Aware DM

In this section, I show how the results in main text hold in the “aware” case. Throughout the section, I assume that the preliminary information perfectly reveals the realization of the signal *i.e.*, I focus on the case where g is degenerate.

First, let me formally define the “aware” case. When the DM forms his belief with his memory $\mathcal{M}_t$, he rationally infers information about the signals in periods $\{t' \mid t' \notin \mathcal{M}_t\}$ from the fact that he had chosen to ignore them. More specifically, he forms a conjecture $\tilde{\sigma}_{t'}$ about his processing strategies in those periods t' and his posterior is given by the following Bayesian formulas:

$$\begin{aligned} p_L^t(\mathcal{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}) &= \left(1 + \frac{p_R}{p_L \prod_{t' \in \mathcal{M}_t} s_{t'} \prod_{t' \notin \mathcal{M}_t} \mathcal{R}_{t'}(\tilde{\sigma}_{t'})} \right)^{-1}; \\ p_R^t(\mathcal{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}) &= 1 - \text{Pr}_L(\mathcal{M}_t, (\tilde{\sigma}_{t'})_{t'=1}^{t-1}), \end{aligned}$$

where $\mathcal{R}_{t'}(\tilde{\sigma}_{t'})$ is the ratio of the conjecture probability of ignoring a period t' signal in state L over state R :

$$\mathcal{R}_{t'}(\tilde{\sigma}_{t'}) = \frac{\int (\mathbb{1}_{\tilde{\sigma}_{t'}(s_{t'}, \mathcal{M}_{t'})=I}) f_{t'L}(s_{t'}) ds_{t'}}{\int (\mathbb{1}_{\tilde{\sigma}_{t'}(s_{t'}, \mathcal{M}_{t'})=I}) f_{t'R}(s_{t'}) ds_{t'}}.$$

On the other hand, when he decides whether to process or ignore a signal, he knows that if he ignores it, his future selfs will rationally infer information about the signal. The game is a multi-self dynamic game with incomplete information. The solution concept used is therefore perfect Bayesian Nash equilibrium, which requires the optimality of the processing strategies as well as the consistency of the DM’s conjecture, *i.e.*, $\tilde{\sigma}_{t'}$ has to coincide with the equilibrium processing strategy for all $t' = 1, \dots, T$.

First, I analyze the case where $\bar{T} > 1$. As in section 5 of the main text, I analyze the processing strategy at period t , taking $\text{Pr}_{\mathcal{M}_t}^t(a \mid \omega)$ and

¹University of Cambridge. Email: btkl3@cam.ac.uk

$\Delta_{s_t, \mathcal{M}_t}^t(a \mid \omega)$ as given. Note that two functions do not specify how the DM forms his posterior belief, and therefore encompass both the aware and unaware case. Therefore, lemma 1 holds and the processing strategy of the DM is “asymmetric”. Moreover, as the definitions of “awareness” and “unawareness” make no differences when the DM processes all signals, proposition 1 also holds, *i.e.*, the DM processes almost all signals in equilibrium when $\bar{T} \geq T$ but that is not true when $\bar{T} < T$.

Now I turn to the case where $\bar{T} = 1$. As in the unaware case, it is without loss of generality to further simplify the model to $T = 2$. The equivalent result is presented in appendix C. Different from the unaware case, there is no guarantee that there exists a unique equilibrium in the aware case. The following proposition proves the existence of a perfect Bayesian Nash equilibrium.

Proposition A.1. *There exists a perfect Bayesian Nash equilibrium in the aware case.*

Proof. By proposition C.2, it is sufficient to prove that there exists a perfect Bayesian Nash equilibrium for $T = 2$. First note that the equilibrium strategy is a threshold strategy, with thresholds (Φ^+, Φ^-) . Denote $\mathcal{R}_1$ as

$$\mathcal{R}_1 = \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}.$$

As $\Phi^+ \geq \frac{u_R p_R}{u_L p_L}$ and $\Phi^- > \frac{u_L p_L}{u_R p_R}$, $\mathcal{R}_1 \in \left(\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)} \right)$. Given the optimality of the processing strategy, the perfect Bayesian Nash equilibrium is a fixed point of the following equation:

$$\mathcal{R}_1 = \frac{F_{1L} \left(\frac{u_R p_R}{u_L p_L} \frac{F_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{F_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right) - F_{1L} \left(\frac{u_R p_R}{u_L p_L} \frac{1 - F_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{1 - F_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right)}{F_{1R} \left(\frac{u_R p_R}{u_L p_L} \frac{F_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{F_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right) - F_{1R} \left(\frac{u_R p_R}{u_L p_L} \frac{1 - F_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)}{1 - F_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1} \right)} \right)}$$

Denote the equation as $\mathcal{R}_1 = \psi(\mathcal{R}_1)$. The mapping ψ is continuous mapping from $\left[\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)} \right]$ to itself, which is a convex and compact set. By the Brouwer’s fixed point theorem, there exists a fixed point of the equation which belongs to the set $\left[\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1-F_{1L}(1)}{1-F_{1R}(1)} \right]$. As the two end points $\frac{F_{1L}(1)}{F_{1R}(1)}$ and $\frac{1-F_{1L}(1)}{1-F_{1R}(1)}$ are not a fix point of the equation, there exists a fixed point $\mathcal{R}_1^*$ such

that

$$\begin{aligned}\mathcal{R}_1^* &\in \left(\frac{F_{1L}(1)}{F_{1R}(1)}, \frac{1 - F_{1L}(1)}{1 - F_{1R}(1)} \right); \\ \mathcal{R}_1^* &= \psi(\mathcal{R}_1^*).\end{aligned}$$

□

The proof follows classical fixed point arguments. Note that there may exist multiple equilibria because of the self-fulfilling nature of the incomplete information game. If the future selfs of the DM conjecture that his period t self ignores only belief-challenging information, his period t self will have more incentive to ignore belief-challenging information as he knows that his future selfs will rationally infer information from it. On the other hand, he also has more incentive to process belief-confirming information because otherwise, his future selfs will mistakenly conjecture that he has received a belief-challenging information in period t .

In the following, I show that the behavioral implications presented in section 6 of the main text hold qualitatively for all equilibria in the aware case. First, I characterize the equilibrium processing strategy, as in proposition 2.

Proposition A.2. *The equilibrium processing strategy of the aware DM is as follows:*

- *he processes a belief confirming signal $s_1 > 1$ if and only if*

$$STR(s_1) = s_1 \geq \Phi^+;$$

- *he processes a pure noise or a weak belief-challenging signal $s_1 \in [\frac{u_R p_R}{u_L p_L}, 1]$ if and only if*

$$s_1 > \Phi^+;$$

- *he processes a strong belief-challenging signal $s_1 < \frac{u_R p_R}{u_L p_L}$ if and only if*

$$STR(s_1) = s_1^{-1} > \Phi^-,$$

where Φ^+ and Φ^- are the fixed point of the following system of equations:

$$\begin{aligned}\Phi^+ &= \frac{u_R p_R}{u_L p_L} STR \left(s_2 \in \left(0, \frac{u_R p_R}{u_L p_L \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}} \right) \right) \\ \Phi^- &= \frac{u_L p_L}{u_R p_R} STR \left(s_2 \in \left[\frac{u_R p_R}{u_L p_L \frac{F_{1L}(\Phi^+) - F_{1L}(1/\Phi^-)}{F_{1R}(\Phi^+) - F_{1R}(1/\Phi^-)}}, \infty \right) \right)\end{aligned}\tag{A.1}$$

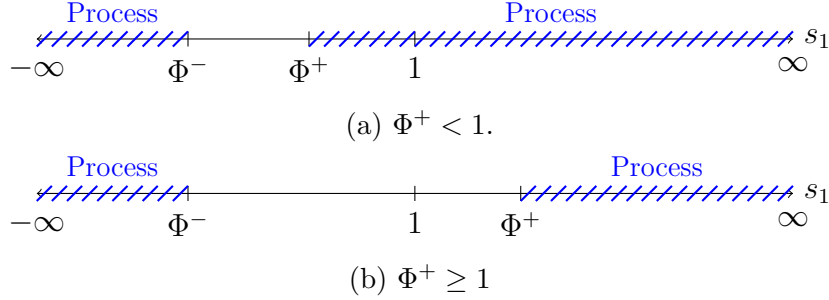


Figure 1: Equilibrium Information Processing Strategy: the DM processes s_1 if and only if its realization falls on the shaded area.

The proof is very similar to that of proposition 2 and is therefore skipped. The result is illustrate in figure 1. Note that different from the unaware case, the DM may process weak belief-challenging information (when $\Phi^+ < 1$)². This difference is again driven by the self-fulfilling nature of the game: suppose the DM's period 2 self conjectures that his period 1 self ignores strong belief-challenging signals but processes weak belief-challenging signals. In the view of his period 1 self, he knows that if he ignores the weak belief-challenging signals, his period 2 self will mistakenly infer that s_1 strongly supports state R . It gives the DM's period 1 self incentive to process weak belief-challenging signals and take action l , as otherwise he will be underconfident about state L and switch to action r too easily in period 2.

Despite the slight difference, the behavior implications hold qualitatively. First, the DM processes strong enough signals, but may ignore weak signals, *i.e.*, it resembles a preference for strong signals (although it is weaker than the version in the unaware case). Moreover, by equation (A.1), the limits of Φ^+ and Φ^- when $u_L p_L / u_R p_R \rightarrow +\infty$ are the same as in the unaware case³. Hence, when $u_L p_L / u_R p_R$ is large enough, $\Phi^- > \Phi^+$. The results are summarized in the following proposition.

Proposition A.3. *The equilibrium processing strategy of the DM explains the following behavioral phenomena:*

²Note that Φ^- is always below 1 but Φ^+ could be either bigger than or smaller than 1. It is because of the asymmetry of the set of signals inducing the two different actions: processing $s_1 \geq \frac{u_R p_R}{u_L p_L}$ induces action l while processing $s_1 < \frac{u_R p_R}{u_L p_L}$ induces action r .

³That is, as shown in the proof of proposition 3,

$$\lim_{u_L p_L / u_R p_R \rightarrow +\infty} \Phi^+ = 1$$

$$\lim_{u_L p_L / u_R p_R \rightarrow +\infty} \Phi^- = +\infty$$

1. **(Preference for strong signals)**

The DM processes a signal s_1 if it is strong enough.

2. **(Confirmation bias for confident individuals)**

If the DM a priori believes strongly that state L is true, his processing strategy exhibits confirmation bias, i.e., $\Phi^- > \Phi^+$ if p_L is large enough.

3. **(Wishful thinking if one state is much more desirable than the others)**

If state L is much more desirable than state R , the DM's processing strategy exhibits confirmation bias, i.e., $\Phi^- > \Phi^+$ if u_L/u_R is large enough.

Note that the results hold in all equilibria in the aware case, i.e., all equilibrium processing strategies of the DM have (qualitatively) the same behavioral implications as in the unaware case.

Lastly, because there is an issue of multiple equilibria, the result of the comparative analysis would not be as clean as in the unaware case. However, the insights still hold true. As shown in equation (A.1), the thresholds which characterize the equilibrium processing strategy depend only on a subset of all realizations of s_2 . As in the unaware case, providing in average more informative signal to the DM does not necessarily increase the average strength of a subset of all realizations of s_2 . Therefore, it could strengthen the confirmation bias of the DM.

B Optimality of the processing strategy

Before I present the omitted proofs in the main text, I now discuss the optimality of the equilibrium processing strategy. In the main text, the DM decides whether to process or ignore the signal given the preliminary information about its realization. Therefore, the equilibrium processing strategy $\sigma_t(g, \mathcal{M}_t)$ is optimal all any preliminary information g . Moreover, as it maximizes the DM's expected utility for all possible preliminary information in each period, it also maximizes his expected ex-ante utility in every period t , which gives the following result.

Proposition B.1. *The processing strategy characterized by lemma 1 in the main text maximizes the expected utility of the DM's period t self.*

In the main text, I show that the optimal processing strategy explains some well-documented behavioral phenomena in the presence of limitation in processing ability. The optimality is achieved as the DM evaluates the

expected utility of processing and ignoring a signal given his knowledge of the signal realization. In contrast, proposition B.1 suggests that the results also hold if we consider strategies that maximize the DM's expected utility at the beginning of each period. This result is also useful for the proof for the equivalence result, as shown in the next section.

C Equivalence Results

In this section, I present two equivalence results which allow me to simplify the model with $T > 2$ and $\bar{T} = 1$ to a model with $T = 2$ and $\bar{T} = 1$. The idea is to show that the equilibrium processing strategy at any period $t < T$ in a model with $T > 2$ is equivalent to the equilibrium processing strategy at period 1 in the simplified model with $T = 2$. I first present the result in the unaware case.

C.1 Unaware Case

Proposition C.1. *There exists some p.d.f. $(\tilde{f}_{2L}, \tilde{f}_{2R})$ such that the following two equilibrium strategies are equivalent:*

1. *the equilibrium processing strategy of the DM with belief (p_L^t, p_R^t) at period $t < T$, under a setting with $T > 2$, $\bar{T} = 1$ and information structure $\{(f_{t'L}, f_{t'R})\}_{t'=t+1}^T$;*
2. *the equilibrium processing strategy of the DM with prior belief $(p_L, p_R) = (p_L^t, p_R^t)$ at period 1, under a setting with $T = 2$, $\bar{T} = 1$ and information structure $(\tilde{f}_{2L}, \tilde{f}_{2R})$.*

Proof. I prove the proposition under the assumption the preliminary information perfectly reveals the signal realization. The proof is similar in the general setting where g could be degenerate⁴. Note that with $\bar{T} = 1$, the memory of the DM is always $\emptyset$ before processing a signal. Thus, I simply use $\sigma_t(s_t)$ to denote the processing strategy of the DM at period t . First consider the first point where $T > 2$ and without loss of generality assume $u_L p_L^t \geq u_R p_R^t$. By lemma 1 in the main text, the decision maker processes

⁴The proof is identical if I replace $\mathbb{1}_{\sigma_h(s)=P}$ by $\int \mathbb{1}_{\sigma_h(s)=P} g(s) dg$ and $\mathbb{1}_{\sigma_h(s)=I}$ by $\int \mathbb{1}_{\sigma_h(s)=I} g(s) dg$.

signal $s_t \geq \frac{u_R p_R^t}{u_L p_L^t}$ if and only if

$$\begin{aligned}
s_t &\geq \frac{u_R p_R^t}{u_L p_L^t} \frac{\Delta_{s_t, \mathcal{M}_t}^t(r | R)}{\Delta_{s_t, \mathcal{M}_t}^t(r | L)} \\
&= \frac{u_R p_R^t}{u_L p_L^t} \frac{\sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds}{\sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds}.
\end{aligned} \tag{C.1}$$

On the other hand, he processes signal $s_t < \frac{u_R p_R^t}{u_L p_L^t}$ if and only if

$$\begin{aligned}
s_t^{-1} &> \frac{u_L p_L^t}{u_R p_R^t} \frac{\Delta_{s_t, \mathcal{M}_t}^t(l | L)}{\Delta_{s_t, \mathcal{M}_t}^t(l | R)} \\
&= \frac{u_L p_L^t}{u_R p_R^t} \frac{1 - \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kL}(s) ds}{1 - \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kR}(s) ds}.
\end{aligned} \tag{C.2}$$

Now consider point 2 where $T = 2$ with information structure $(\tilde{f}_{2L}, \tilde{f}_{2R})$ and prior belief $p_L = p_L^t$. By lemma 1 in the main text, the decision maker processes signal $s_1 \geq \frac{u_R p_R}{u_L p_L}$ if

$$s_t \geq \frac{u_R p_R}{u_L p_L} \frac{\tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L} \right)}{\tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L} \right)}, \tag{C.3}$$

and he processes signal $s_1 < \frac{u_R p_R}{u_L p_L}$ if and only if

$$s_t^{-1} > \frac{u_L p_L}{u_R p_R} \frac{1 - \tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L} \right)}{1 - \tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L} \right)}. \tag{C.4}$$

By comparing equation (C.1) to equation (C.3), and equation (C.2) to equation (C.4), the equilibrium processing strategy in the two cases are equivalent if and only if

$$\begin{aligned}
\tilde{F}_{2L}\left(\frac{u_R p_R}{u_L p_L}\right) &= \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hL} ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kL}(s) ds; \\
\tilde{F}_{2R}\left(\frac{u_R p_R}{u_L p_L}\right) &= \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h^*(s')=I}) f_{hR} ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t}} \mathbb{1}_{\sigma_k^*(s)=P} f_{kR}(s) ds.
\end{aligned} \tag{C.5}$$

To prove that there exists p.d.f. $\tilde{f}_{2L}$ and $\tilde{f}_{2R}$ that generates the c.d.f.s evaluated at $\left(\frac{u_R p_R}{u_L p_L}\right)$ with values defined in equation (C.5), it remains to prove that

$$\frac{\tilde{F}_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{\tilde{F}_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} > \frac{u_L p_L}{u_R p_R}.$$

Note that by definition, equation (C.5) implies that

$$\frac{\tilde{F}_{2R}\left(\frac{u_R p_R}{u_L p_L}\right)}{\tilde{F}_{2L}\left(\frac{u_R p_R}{u_L p_L}\right)} = \frac{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid R)}{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid L)}.$$

On the other hand, by proposition B.1, the expected utility of the DM at the beginning of period $t+1$ is weakly greater than that if he chooses to process all s_t :

$$\begin{aligned}
&u_L p_L (1 - \Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid L)) + u_R p_R \Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid R) \\
&\geq u_L p_L \left(1 - F_{(t+1)L}\left(\frac{u_R p_R}{u_L p_L}\right)\right) + u_R p_R F_{(t+1)R}\left(\frac{u_R p_R}{u_L p_L}\right) \\
&> u_L p_L,
\end{aligned}$$

where the last inequality is implied by the fact receiving one signal always improves expected utility, in comparison to receiving no signal. Rearranging gives:

$$\begin{aligned}
u_R p_R \Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid R) &> u_L p_L \Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid L); \\
\frac{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid R)}{\Pr_{\mathcal{M}_{t+1}}^{t+1}(r \mid L)} &> \frac{u_L p_L}{u_R p_R}.
\end{aligned}$$

The results follow. □

C.2 Aware Case

Proposition C.2. *There exists some p.d.f. $(\tilde{f}_{1L}, \tilde{f}_{1R}), (\tilde{f}_{2L}, \tilde{f}_{2R})$ such that the following two sets of equilibrium strategies are equivalent:*

1. *the set of equilibrium processing strategies of the DM with belief (p_L^t, p_R^t) at period $t < T$, under a setting with $T > 2$, $\bar{T} = 1$ and information structure $\{(f_{t'L}, f_{t'R})\}_{t'=t}^T$ (assuming that it exists);*
2. *the set of equilibrium processing strategies of the DM with prior belief $(p_L, p_R) = (p_L^t, p_R^t)$ at period 1, under a setting with $T = 2$, $\bar{T} = 1$ and information structure $(\tilde{f}_{1L}, \tilde{f}_{1R}), (\tilde{f}_{2L}, \tilde{f}_{2R})$.*

Proof. First consider point 1 where $T > 2$ and without loss of generality assume $u_L p_L^t \geq u_R p_R^t$. For simplicity and with a bit of abuse in notations, define $\mathcal{R}_{t'}^* = \prod_{t=1}^{t'-1} \mathcal{R}_t(\tilde{\sigma}_t)$, which is the conjectured ratio of ignoring all the previous signals in state L over than in state R . Note that at the beginning of period $t' > t$, the expected utility of action l over that of action r equals $\frac{u_L p_L^t \mathcal{R}_{t'}^*}{u_R p_R^t}$. By lemma 1 in the main text, the decision maker processes signal $s_t \geq \frac{u_R p_R^t}{u_L p_L^t}$ if and only if

$$\begin{aligned} s_t &\geq \frac{u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(r | R)}{u_L p_L^t \Delta_{s_t, \mathcal{M}_t}^t(r | L)} \\ &= \frac{u_R p_R^t \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds}{u_L p_L^t \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds} \end{aligned} \quad (\text{C.6})$$

On the other hand, he processes signal $s_t < \frac{u_R p_R^t}{u_L p_L^t}$ if and only if

$$\begin{aligned} s_t^{-1} &> \frac{u_L p_L^t \Delta_{s_t, \mathcal{M}_t}^t(l | L)}{u_R p_R^t \Delta_{s_t, \mathcal{M}_t}^t(l | R)} \\ &= \frac{u_L p_L^t \left[1 - \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds \right]}{u_R p_R^t \left[1 - \sum_{k=t+1}^T \left[\mathbb{1}_{k>t+1} \prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' + \mathbb{1}_{k=t+1} \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds \right]} \end{aligned} \quad (\text{C.7})$$

The equilibrium strategy is given by the equation (C.6) and (C.7). Now

define $\tilde{s} = s \frac{\mathcal{R}_k^*}{\mathcal{R}_{t+1}^*}$, note that

$$\begin{aligned}
& \frac{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kR}(s) ds}{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_k^*}} \mathbb{1}_{\sigma_k(s)=P} f_{kL}(s) ds} \\
&= \frac{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_{t+1}^*}} \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}}{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_{t+1}^*}} \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}} \\
&= \frac{\int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_{t+1}^*}} \sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}}{\int_0^{\frac{u_R p_R^t}{u_L p_L^t \mathcal{R}_{t+1}^*}} \sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*) d\tilde{s}}.
\end{aligned}$$

Now I define $(\tilde{f}_{2L}, \tilde{f}_{2R})$ as follows and verify whether it is a probability distribution function and whether there exists $(\tilde{f}_{1L}, \tilde{f}_{1R})$ such that the equilibrium processing strategy of the DM in period $t = 1$ where $T = 2$ is equivalent to that characterized in equation (C.6) and (C.7):

$$\begin{aligned}
\tilde{f}_{2L}(\tilde{s}) &= \sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*); \\
\tilde{f}_{2R}(\tilde{s}) &= \sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*).
\end{aligned}$$

First, it is clear that $\int \tilde{f}_{2L}(\tilde{s}) d\tilde{s} = \int \tilde{f}_{2R}(\tilde{s}) d\tilde{s} = 1$ as the DM always processes one signal in equilibrium. On the other hand,

$$\begin{aligned}
\frac{\tilde{f}_{2L}(\tilde{s})}{\tilde{f}_{2R}(\tilde{s})} &= \frac{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hL}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*)}{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)} \\
&= \frac{\sum_{k=t+1}^T \mathcal{R}_k^* \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kL}(\tilde{s}/\mathcal{R}_k^*)}{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)} \\
&= \frac{\sum_{k=t+1}^T \tilde{s} \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)}{\sum_{k=t+1}^T \left[\prod_{h=t+1}^{k-1} \int_0^\infty (\mathbb{1}_{\sigma_h(s')=I}) f_{hR}(s') ds' \right] \mathbb{1}_{\sigma_k(\tilde{s}/\mathcal{R}_k^*)=P} f_{kR}(\tilde{s}/\mathcal{R}_k^*)} \\
&= \tilde{s}.
\end{aligned}$$

Denote $(\tilde{F}_{2L}, \tilde{F}_{2R})$ as the c.d.f. associated with $(\tilde{f}_{2L}, \tilde{f}_{2R})$. In point 2 where $T = 2$, there exists $(\tilde{f}_{1L}, \tilde{f}_{1R})$ such that the DM processes signal $s_1 \geq \frac{u_R p_R}{u_L p_L}$ if and only if

$$s_1 \geq \frac{u_R p_R}{u_L p_L} \frac{\tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{\tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)},$$

and processes signal $s_t < \frac{u_R p_R}{u_L p_L}$ if and only if

$$s_1^{-1} \geq \frac{u_L p_L}{u_R p_R} \frac{1 - \tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{1 - \tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}.$$

where $(\tilde{F}_{1L}, \tilde{F}_{1R})$ satisfy

$$\mathcal{R}_1^* = \frac{\tilde{F}_{1L} \left(\frac{u_R p_R}{u_L p_L} \frac{\tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{\tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)} \right) - \tilde{F}_{1L} \left(\frac{u_L p_L}{u_R p_R} \frac{1 - \tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{1 - \tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)} \right)}{\tilde{F}_{1R} \left(\frac{u_R p_R}{u_L p_L} \frac{\tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{\tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)} \right) - \tilde{F}_{1R} \left(\frac{u_L p_L}{u_R p_R} \frac{1 - \tilde{F}_{2L} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)}{1 - \tilde{F}_{2R} \left(\frac{u_R p_R}{u_L p_L \mathcal{R}_1^*} \right)} \right)}.$$

The result follows. □